The Economic Impact of Home Building in Washington:

Comparing Costs to Revenue for State and Local Governments

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Housing Policy Department

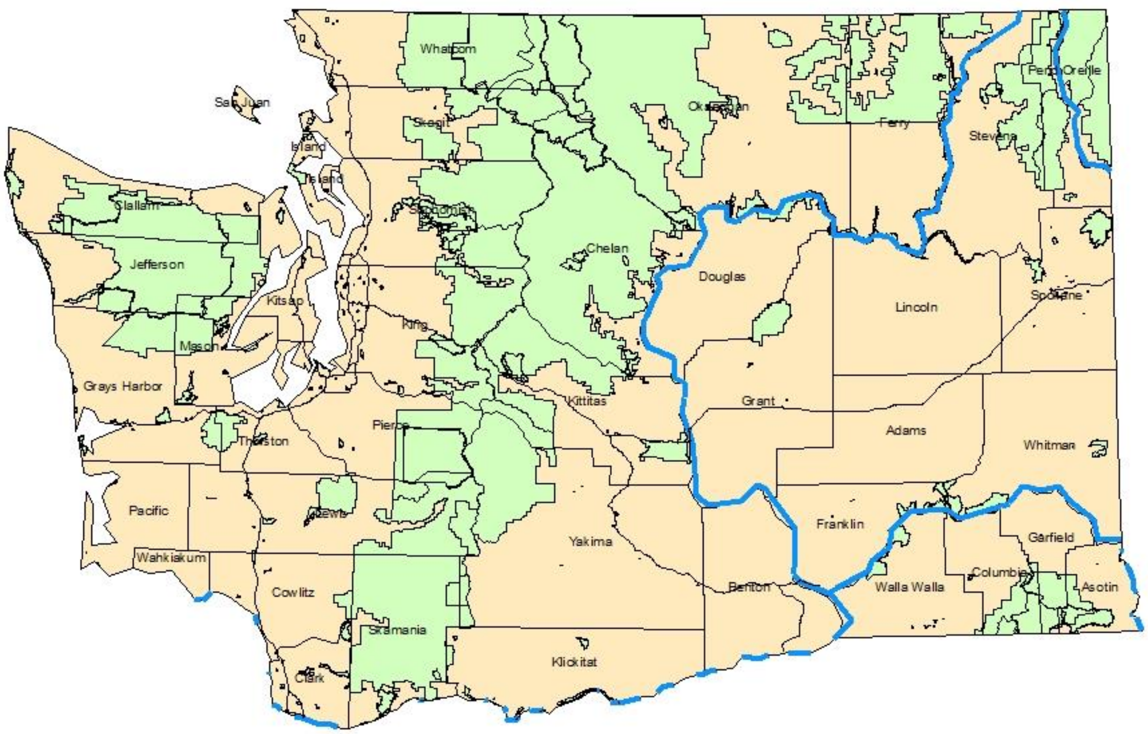
National Association of Home Builders

Introduction

Home building generates economic impacts in the state where it takes place, including income and jobs for residents of the state, and revenue for the state government and local governments within the state. It also typically imposes costs on state and local governments—such as the costs of providing primary and secondary education, police and fire protection, and water and sewer service. Not only do these services require annual expenditures for items such as teacher salaries, they typically also require capital investment in buildings, other structures, and equipment that state and local governments within the state own and maintain.

This report presents estimates of the impacts of building 100 single-family homes in the State of Washington (Figure 1).

Figure 1. State of Washington



The benefits generated by this level of home construction activity are reported in a separate NAHB document.[1] This report presents estimates of the costs—including current and capital expenses—that new homes impose on jurisdictions in the area and compares those costs to the revenue generated. The results are intended to answer the question of whether or not, from the standpoint of state and local governments, residential development pays for itself—and, if so, how quickly.

[1] "The Economic Impact of Home Building in Washington: Income, Jobs, and Taxes Generated," completed by NAHB in August 2021.

Costs Compared to Revenue

This section summarizes the cost-revenue comparisons. The relevant assumptions about the single-family homes built in the State of Washington (including their average price, property tax payments, and construction-related fees incurred) are described in the NAHB report, *The Economic Impact of Home Building in Washington: Income, Jobs and Taxes Generated*.

- In the first year, the 100 single-family homes built in the State of Washington result in an estimated
  - **$10.43 million** in tax and other revenue for the state government as well as local governments in the state,[2]
  - **$739,000** in current expenditures by the state and local governments to provide public services to the net new households at current levels, and
  - **$1.33 million** in capital investment for new structures and equipment undertaken by the state and local governments

  The analysis assumes that state and local governments finance the capital investment by borrowing at the current rate of 3.76 percent on tax-exempt bonds.[3]

- In a typical year after the first, the 100 single-family homes result in
  - **$1.96 million** in tax and other revenue for the state and local governments in Washington and
  - **$1.48 million** in state and local government expenditures to continue providing services at current levels.

- The difference between government revenue and current expenditures is defined as an "operating surplus." In this case, the first-year operating surplus is large enough to service and pay off all incurred by investing in structures and equipment by the end of the first year. After that, the operating surpluses will be available to finance other projects or reduce taxes. After 15 years, the homes will generate a cumulative **$37.9 million in revenue** compared to **$22.9 million in costs**, including annual current expenses, capital investment, and interest on debt (Figure 2).

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[2] This assumes that homes are occupied at a constant rate during the year, so that the year captures one-half of the ongoing, annual revenue generated as the result of increased property taxes and the new residents participating in the economy of Washington.

[3] The analysis assumes that there is currently no excess capacity, that the state government, as well as local governments within the state, invest in capital before the homes are built, and that no fees or other revenue generated by construction activity are available to finance the investment, so that all capital investment at the beginning of the first year is financed by debt. This is a conservative assumption that results in an upper bound estimate on the costs incurred by state and local governments. The particular interest rate is based on the Bond Buyer Municipal Bond Index, which is based on prices for 40 long-term municipal bonds.

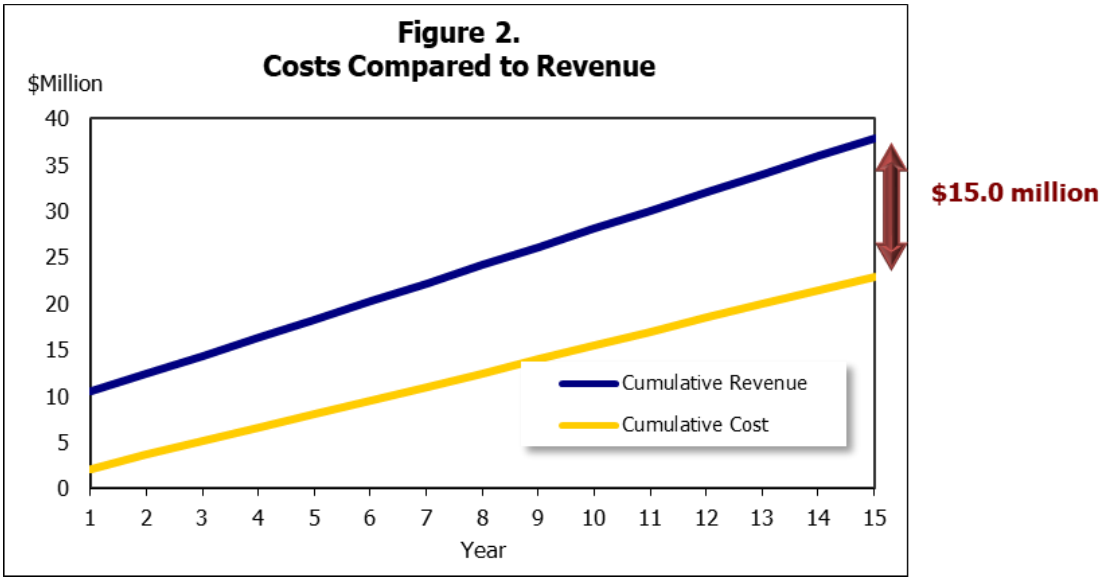

Figure 2.
Costs Compared to Revenue

Method Used to Estimate Costs

The method for estimating state and local government revenue generated by home building is explained in the attachment to *The Economic Impact of Home Building in Washington: Income, Jobs and Taxes Generated*. This section describes how costs are estimated.

The general approach is to assume local jurisdictions supply residents of new homes with the same services that they currently provide, on average, to occupants of existing structures. The amount that any jurisdiction spends is available from the Census of Governments, where all units of government in the U.S. report line item expenses, revenues, and intergovernmental transfers once every five years to the Governments Division of the U.S. Census Bureau. Census of Governments accounts can be aggregated for every local government in Washington, as well as for the Washington State government, and then used to produce total annual expenses per housing unit (Table 1).

Not surprisingly, cost per housing unit varies substantially across the major service categories. Education accounts for the largest share of annual expenses, followed by the shares for publicly provided health services and miscellaneous functions of general government.

Table 1. Total Annual State and Local Government Expenses per Single-family Housing Unit

Education	$5,928
Police Protection	$555
Fire Protection	$377
Corrections	$578
Streets and Highways	$342
Water Supply	$164
Sewerage	$150
Health Services	$2,325
Recreation and Culture	$373
Other General Government	$2,760
Electric Utilities	$811
Gas Utilities	$1
Public Transit	$225
Other Government Enterprises	$182
Total	$14,772

In deriving the above estimates, water supply and sewerage expenses are allocated based on gallons of water consumed per day by single-family and multifamily households. Streets and highway expenses are allocated based on average number of vehicle trips generated on weekdays. Education is allocated based on average number of public school children age 5 through 18. The remaining expenses listed in Table 1 are assumed to be proportional to household size and are allocated to single-family and multifamily units based on average number of persons per household.[4]

There are several factors present in most parts of the country that tend to reduce education expenses per housing unit. The first is the average number of children going to public schools present in the units. According to the American Community Survey, there is, on average, only a little over one public school child for every three households in the U.S. The number is about 0.4 per household for single-family and under 0.2 per household for multifamily. So, education costs per housing unit are lower than costs per pupil, simply because there is less than one pupil per household.

[4] Information about vehicle trips comes from *Trip Generation Manual, 10th Ed.*, September 2017, Institute of Transportation Engineers: https://www.ite.org/tripgeneration/index.asp. Information about water consumption comes from *Water Demand Trends in the Multifamily Housing Sector,* a study undertaken in 2017 by Jack Kiefer and Lisa Krentz for the Water Research Foundation http://www.waterrf.org/Pages/Index3.aspx. Information about household size and number of public school children comes from the 2016 Public Use Microdata Sample of the American Community Survey, U.S. Census Bureau: https://www.census.gov/programs-surveys/acs/.

In addition to current expenses, providing services to residents requires that local governments make capital expenditures for items such as schools and other buildings, equipment, roads, and other structures.

The process employed by NAHB to estimate capital costs involves several steps. The general approach is to apply parameters from a conventional economic model (a production relationship, where costs are expressed as a function of labor and capital) estimated with state level data to information for a specific local area. State and local government capital in each state can be derived through a procedure that has been established over several decades in the technical literature on public finance (see the technical appendix for details). The parameter estimates are then applied to a local area, where information is available for every variable except capital. The local capital stock then emerges as a residual in the calculation. Consistent with the approach used to estimate current expenses, the amount of capital in each category is expressed as the amount necessary to accommodate an average single-family or average multifamily housing unit (Table 2).

Table 2. State and Local Government Capital per Single-family Housing Unit

Schools	$28
Hospitals	$540
Other Buildings	$623
Highways and streets	$7,377
Conservation & development	$11
Sewer systems	$2,671
Water supply	$1,095
Other structures	$120
Equipment	$812
Total	$13,277

To implement these numbers, several conservative assumptions are made to avoid understating the costs. For example, it is assumed that none of the demand for state and local government capital can be satisfied through current excess capacity. Instead, the state government and local governments in the state invest in new structures and equipment at the start of the first year, before any homes are built. To the extent that this is not true—that, for instance, some revenue from impact or other fees is available to fund part of the capital expenditures—interest costs would be somewhat lower than reported here.

To compare the streams of costs and revenues over time, the analysis assumes that half of the current expenses and half of the ongoing, annual revenues are realized in the first year. This would be the case if construction and occupancy took place at an even rate throughout the year. Revenues in the first year also include all of the one-time construction impacts such as impact and permit fees.

The difference between revenues and current expenses in a given year is an operating surplus. At the start of the first year, capital investment is financed through debt by borrowing at the current tax-exempt bond interest rate,[5] and the interest accrues throughout the year. Each year after that, the operating surplus is used first to pay the interest on the debt, if any exists, then to pay off the debt at the end of the year. Results for the 100 single-family homes built in the State of Washington are shown in Table 3:

Table 3. Results for 100 Single-family Homes Built in the State of Washington

Year	Current Expenses	Revenue	Operating Surplus	Capital Investment Start of Year	Debt Outstanding End of Year	Interest on the Debt	Revenue Net of Costs and Interest
1	738,600	10,430,500	9,691,900	1,327,700	0	49,900	8,314,300
2	1,477,100	1,961,100	484,000	0	0	0	484,000
3	1,477,100	1,961,100	484,000	0	0	0	484,000
4	1,477,100	1,961,100	484,000	0	0	0	484,000
5	1,477,100	1,961,100	484,000	0	0	0	484,000
6	1,477,100	1,961,100	484,000	0	0	0	484,000
7	1,477,100	1,961,100	484,000	0	0	0	484,000
8	1,477,100	1,961,100	484,000	0	0	0	484,000
9	1,477,100	1,961,100	484,000	0	0	0	484,000
10	1,477,100	1,961,100	484,000	0	0	0	484,000
11	1,477,100	1,961,100	484,000	81,200	0	0	402,800
12	1,477,100	1,961,100	484,000	0	0	0	484,000
13	1,477,100	1,961,100	484,000	0	0	0	484,000
14	1,477,100	1,961,100	484,000	0	0	0	484,000
15	1,477,100	1,961,100	484,000	0	0	0	484,000

In this case, revenue net of costs and interest is positive every year, beginning with the first. In fact, revenue net of costs and interest is sufficient to pay off all debt by the end of year one. After that, revenue net of costs generated by the 100 single-family homes is roughly $484,00 per year.

Revenue net of costs and interest falls slightly and temporarily in year 11, due to a cost that local governments incur at that time as capital equipment purchased at the start of the first year becomes fully depreciated and needs to be replaced. All other capital investment consists of structures of various types, and the effective service life for any type of structure is considerably longer than a single decade.

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[5]The interest rate on municipal bonds is the monthly Bond Buyer long-term Municipal Bond Index available on the Bond Buyer Web site: https://data.bondbuyer.com/MarketStatisticsArchive/Search_MBI/11?Name=Municipal%20Bond%20Index

Comparing Costs to Revenue for Local Governments

Technical Appendix on Estimating Capital Owned and Maintained by Local Governments

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Technical Appendix on Estimating Local Capital Owned and Maintained by Local Governments

This appendix explains the method used to estimate the age and dollar value of local government capital by function (education, water and sewer services, etc.). The general approach is to estimate economic relationships using state-level data and then apply parameters from the state-level estimates to local data.

First, a cost share equation based on conventional production theory is described for the structures associated with each function of government. In the equations age of capital is used as a proxy for technologic change. Age of capital, in turn, is estimated as a function of population growth.

The following derivations apply to any one of the ten categories of state and local government capital—e.g., highways or school buildings—tracked in the Bureau of Economic Analysis (BEA) wealth data files. For simplicity, the notation suppresses an explicit reference to capital type. In cases where some detail of the model pertains to a particular type of capital or function of local governments, the text will make that clear.

Let y = output; L= labor, w = the price of labor, and r = the price of capital, and consider a general translog cost function:[6]

(1) $$c_{it} = \beta_0 + \beta_w \ln w_{it} + \beta_r \ln r_{it} + \beta_y \ln y_{it} + \beta_a a_{it} + \tfrac{1}{2}\beta_{ww} (\ln w_{it})^2 + \beta_{wr} \ln w_{it} \ln r_{it} + \tfrac{1}{2}\beta_{rr} (\ln r_{it})^2 + \beta_{wy} \ln w_{it} \ln y_{it} + \beta_{ry} \ln r_{it} \ln y_{it} + \beta_{wa} a_{it} \ln w_{it} + \beta_{ra} a_{it} \ln r_{it} + \beta_{yy} (\ln y_{it})^2 + \beta_{ya} a_{it} \ln y_{it} + \beta_{aa} a_{it}^2$$

In the case where the firm is a government, y_{it} is essentially unmeasurable, so it seems reasonable to assume linear homogeneity in output. This simplifies the translog specification considerably:

(2) $$c_{it} = \beta_0 + \beta_w \ln w_{it} + \beta_r \ln r_{it} + \ln y_{it} + \beta_a a_{it} + \tfrac{1}{2}\beta_{ww} (\ln w_{it})^2 + \beta_{wr} \ln w_{it} \ln r_{it} + \tfrac{1}{2}\beta_{rr} (\ln r_{it})^2 + \beta_{wa} a_{it} \ln w_{it} + \beta_{ra} a_{it} \ln r_{it} + \beta_{aa} a_{it}^2$$

Specification (2) still requires an estimate of $\ln y_{it}$. However, application of Shephard's Lemma generates the following two-equation system:

(3) $$s_{L,\,it} = w_{it} L_{it} / c_{it} = \partial \ln c_{it} / \partial \ln w_{it} = \beta_w + \beta_{ww} \ln w_{it} + \beta_{wr} \ln r_{it} + \beta_{wa} a_{it}$$

(4) $$s_{k\,it} = r_{it} k_t / c_{it} = \partial \ln c_{it} / \partial \ln r_{it} = \beta_r + \beta_{wr} \ln w_{it} + \beta_{rr} \ln r_{it} + \beta_{ra} a_{it}$$

By estimating cost shares rather than the cost function itself, the ability to estimate β_0, β_a, and β_{aa} (essentially nuisance parameters) is lost. Also lost is some precision, in the sense that a lower-order approximation is being estimated.[7] The advantage is relief from the need to supply values for the unobservable y_{it}.

[6] See, for example, Walter Diewert and Terry Wales (1987), "Flexible Functional Forms and Global Curvature Conditions," *Econometrica*, 55, 43-68.

[7] See Henri Theil, *The System-Wide Approach to Microeconomics*, University of Chicago Press, 1980, page 151.

Economic theory implies several restrictions.

Symmetry: β_{wr} is the same in both equations
Linear homogeneity in input prices: $\beta_w + \beta_r = 1$; $½\, \beta_{ww} + \beta_{wr} + ½\, \beta_{rr} = 0$; $\beta_{wa} + \beta_{ra} = 0$.

The restrictions are imposed in the usual way. One of the factor prices (w_{it}) is used as a numeraire; and only one share equation ($s_{L,\,it}$) is estimated, leaving parameters of the second, if needed, to be recovered by simple algebra. The resulting estimating equation is

(5) $$s_{L,\,it} = w_{it}\, L_{it} / (w_{it}\, L_{it} + r_{it}\, k_t) = \beta_w + \beta_{wr} \ln (r_{it}/w_{it}) + \beta_{wa}\, a_{it} + \boldsymbol{\beta}_I' \boldsymbol{I}_{it}$$

where $\boldsymbol{I}_{it}$ is a vector of indicator variables that may be added to equations for some government functions to account for outliers among specific states and time periods. More detail is provided when the regression results are discussed.

Model (5) can be estimated with any standard regression package, provided state-level annual data for L, w, and r can be specified. Series beginning in 1987 for the first two are available from the Government Division of the U.S. Census Bureau. For r, standard practice is followed by assuming cost of capital is the sum of three terms: maintenance (meaning, in this case, all non-labor operating costs), interest, and depreciation.

(6) $$r_{it} = x_{it}/k_{it} + \phi_{it} + \xi_t$$

where x_{it} is the difference between total current expenditures and labor costs, ϕ_{it} is an interest rate for appropriate types of tax-exempt public-purpose government bonds, and ξ_t is the national depreciation rate from BEA's wealth accounts.

To estimate the cost share equations, the same annual interest rate series ϕ_t is used for all states. Because the preferred series not available until 1990, two different sources are used to construct the 1987–2001 annual interest rate series ϕ_t. From 1987 through to the end of 1989, the JP Morgan Revenue Bond Index (RBI) is used. The JP Morgan RBI data are monthly. An annual interest rate is constructed by taking the average of the 12 monthly observations for each calendar year.

From 1990 to the present the Merrill Lynch 20 Year AAA GO series is used. The Merrill Lynch data are provided weekly. An annual interest rate is constructed by taking the average of the 52 observations in each calendar year.

To insure that there is no discontinuity in the series, the annual interest rate from the JP Morgan RBI index for the years 1987 1988 and 1989 is multiplied by the average of the annual ratio of the Merrill Lynch 20 Year AAA GO series divided by the JP Morgan RBI index the for the years 1990 to the present. That ratio turned out to be 0.93. The reason the ratio is less than one is largely because the Merrill Lynch index has a duration that is on average 5 years shorter than the JP Morgan RBI Index.

The final index was chosen following consultation with bonds specialists at both JP Morgan and Merrill Lynch. Although there are hundreds of thousands of unique muni-bonds, and most are rarely if ever traded, the experts felt that a 20 year maturity seemed appropriate and that the ML GO AAA series was probably best for this purpose.

In order to make the cost share equations operational, it's necessary to apportion equipment among the other nine types of capital for which it's possible to approximately match capital with expense and employment data by function of government. In general, a year-zero approach is employed, basing the analysis on the ratio of structures to equipment when both are brand new.

Suppressing the cross-sectional (state) subscript, capital k required for a specific local government function is the sum of structures k_s and equipment k_e:

(7) $$k_t = k_{st} + k_{et}$$

where $k_{st} = k_{s0}(1-\xi_s)^{a_s}$, $k_{et} = k_{e0}(1-\xi_e)^{a_e}$

or, equivalently,

(8) $$k_{s0} = k_{st}(1-\xi_s)^{-a_s}, \quad k_{e0} = k_{et}(1-\xi_e)^{-a_e}$$

Brand new equipment is allocated to brand new structures based on the relative total year-zero values of structures. From this, a ratio z can be derived, which will be the same for all local government functions (or structure types):

(9) $$z = k_{e0}/k_{s0} = k_{et}(1-\xi_e)^{-a_e}\, k_{st}^{-1}(1-\xi_s)^{a_s}$$

The average z ratio for 50 states plus the District of Columbia in the most recent year for which we can compute it (1998) is .11642. This number is used below to help derive estimates of government-owned equipment and structures for a particular local area.

The blended ages and depreciation rates for total capital (structures and equipment) were used to compute the independent variables in the estimating equations. The nine equations (one for each function of government) were estimated, using data for the period where complete state-level government employment and finance data were available—1987 through 1998. The procedure converged quickly (in four iterations). Results are shown in Table 3.

Fit of the model was improved by including a number of indicator variables, up to three per equation. These are identified as I1, I2, and I3 in Table A1 and defined in Table A2.

Not all of the cost equations contain an indicator variable, and each indicator captures only a small number of states. Several variables simply indicate that an observation is for the state of Alaska, and it seems reasonable to suppose that the technology of providing some government services in Alaska would be different than in many other states. In the case of housing, New York appears to be an isolated outlier, and again that is not especially surprising. Other indicators capture a small number of states in New England or the Rocky Mountain area. The conservation series showed a clear break between 1991 and 1992 in Arizona. The Census Bureau instituted some procedural changes involving the collection and reporting of government finance data beginning in 1992.

Table A1. Regression Results: Cost Share Equations

	β_w	β_{wr}	β_{wa}	I1	I2	I3	Adj R^2
Residential	-0.5454	-0.1082	0.0051	0.1531	0.2150		.453
	(.0001)	(.0001)	(.0158)	(.0001)	(.0001)		
Education	-0.3801	-0.1391	0.0156				.545
	(.0001)	(.0001)	(.0001)				
Hospital	0.5682	-0.1413	-0.0247	-0.1793			.506
	(.0001)	(.0001)	(.0001)	(.0001)			
Other Buildings	0.3970	-0.1655	-0.0368				.784
	(.0001)	(.0001)	(.0001)				
Streets & Highways	-0.0345	-0.0723	-0.0110	0.2072			.598
	(.4529)	(.0001)	(.0001)	(.0001)			
Conservation	0.1846	-0.0524	-0.0017	0.3443	-0.2017	0.1210	.483
	(.0165)	(.0001)	(.6021)	(.0001)	(.0001)	(.0001)	
Sewer	-0.4148	-0.0861	0.0018				.522
	(.0001)	(.0001)	(.1985)				
Water	-0.0336	-0.1077	-0.0169				.413
	(.5780)	(.0001)	(.0001)				
Other Structures	-0.2342	-0.1112	-0.0111	0.39629			.566
	(.0021)	(.0001)	(.0004)	(.0001)			

Table A2: Indicator Variables for Cost Share Equations

Capital type	Variable	Condition for I=1
Residential	I1	state=AK
	I2	state=NY
Hospital	I1	state=AZ, NH, or VT
Streets & Highways	I1	state=AK
Conservation	I1	state=AK
	I2	state =NY or CT; or state=AZ and year < 1992
	I3	state=ID, MT, ND, or WY
Other Structures	I1	state= NE, NY, or WA

In the equations above, age of the capital stock appears as an explanatory variable. This is not readily available, even at the state level. A commonly used approach employs perpetual accounting, investment, and depreciation rates to base-year estimates.[8] The procedure used here begins with that approach, but then relates the investment rates to population growth rates, one of the few items for which consistent time series are available for individual U.S. counties.

From BEA national wealth data, the following are available or can easily be computed:

ξ = real annual rate of depreciation (defined broadly, as BEA does, to include a normal rate of obsolescence and retirement of assets)
$>$ = monthly depreciation rate, a simple algebraic transformation of ξ.
N_t = real, net (of depreciation) rate of investment in year t, t=1946,…,2000.

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[8] As in Douglas Holtz-Eakin, "State-Specific Estimates of State and Local Government Capital," *Regional Science and Urban Economics*, Vol. 23, No. 2, April 1993, pp. 185-210.

From data compiled by the Governments Division of the Census Bureau, and ratios employed by BEA to analyze this data, the following can be computed for state i and t=1977,…,1999:

vn_{it} = real investment in new assets state i in year t.
ve_{it} = real investment in existing assets state i in year t.
v_{it} = real investment in state i in year $t = vn_{it} + ve_{it}$.
x_{it} = current expenditures associated with the relevant type of capital state i in year t.

From standard Census Bureau data it is possible to compute
Π_{it} = population growth in the state relative to the national rate; i.e.,

$$\Pi_{it} = \frac{\Delta\rho_{it}}{\rho_{it-1}}\left[\frac{\sum_i \Delta\rho_{it}}{\sum_i \rho_{it-1}}\right]^{-1}$$

The starting point consists of initial end-of-year estimates of the real capital stock, k^0_{i76} , determined by allocating capital to each state according to its share of current expenditure, x_{i77}. This procedure, the one employed for example by Holtz-Eakin (1993), is used here only for the purpose of supplying initial values to be modified in subsequent iterations.

Perpetual inventory accounting can be used to calculate the following recursively for t=1977,…,1999:

(10) $$k^0_{i\,t+1} = k^0_{it}(1-\xi) + v_{it+1}(1->)^6$$

This assumes that investment made during period t+1 depreciates an average of 6 months by the end of the period. Then relative (to the national rate) net real rates of investment can also be computed:

(11) $$\equiv^0_{i\,t} = \left[\frac{v_{it} - \delta\, k^0_{it-1}}{k^0_{it-1}}\right] N_t^{-1}$$

The goal is to obtain estimates of parameters $\forall_j$ and 2_q in the following regression relationship:

(12) $$\equiv^0_{i\,t} = \sum_{j=1}^{J} \alpha^0_j \rho^0_{it-j} + \sum_{q=1}^{Q} \vartheta_q D_q$$

where J is the longest lag considered and the D_q are indicator (dummy) variables. The hypothesis underlying this specification is that a state's rate of investment (relative to the national rate) is a function of past rates of its population growth (also relative to the national rate), with indicator variables to account for anomalies in some states due to peculiarities that are difficult to observe and quantify. Inspection of the pair wise correlations between $\equiv_{i\,t}$ and Π_{it-j} reveal that they begin to decline at or before the lag reaches eight years, depending on the type of capital. Thus, model specification for each type of capital began by tentatively considering population growth effects up to J=8. The final specification varies from case to case.

As a practical matter, the final specifications employ averages of population growth rates lagged over several years. Over the course of several experiments, the sum of the coefficients on the population variables never changed substantially when an average was substituted for a series of individual lags. Coefficients on individual lags tended to fluctuate widely and lack statistical significance, due to collinearity. The use of averages thus aids interpretation without impacting the marginal impacts predicted by the equations in a meaningful way.

Three indicator variables were used in all but the hospital capital equation, which employed four. In most cases, indicator variables flag relatively few states (Table A3).

Table A3: Indicator Variables for Relative Investment Rate Equations

Capital Category	DVERYHI=1	DHIGH=1	DLOW=1	DVERYLOW=1
1 Equipment	DC, WY	AZ, CO, MT, UT	AR, NH, RI	
2 Residential Buildings	DC, HI, MA, NY	CT, DE, RI	CO, FL, ID, NM, TX, UT, VT, WY	
3 Educational Buildings	WY	HI, NM, TX	CA, VT, WI	
4 Hospital Buildings	WY	AL, FL, GA, HI, IA, ID, KS, NY, OH, WA	AR, CT, DE, IL, KY, ME, OR, UT, WI, WV	AZ, VT
5 Other Buildings	DC, WY	HI, MD	AR	
6 Highways and Streets	WY	DC, IA, MN, MT, ND, NE	AR, ME, NH, SC, VT	
7 Conservation & Development	HI, WY	AZ, LA, MT	AL, NY, OK, TN, VA	
8 Sewer Systems & Structures	DC, NY, WA	MA, MD, NJ, OH, RI, WI	AR, NC	
9 Water Supply Facilities	CO, DC, SD, WY	FL, NV	DE, NH	
10 Other Structures	DC	NE	NH	

Given initial estimates, it's possible to begin the perpetual inventory accounting process at an earlier date. If we assume that the World War II period was atypical and restrict ourselves to post-war population data, an 8-year lag in (12) implies that 1954 is the first year for which we can obtain state investment estimates. Hence, state capital stocks in 1953 are estimated by allocating the national capital stock in that year according to its share of the U.S. population, then estimating state investment in the years from 1954 through 1976 recursively according to

(13) $$\nu^0_{it} = k^0_{it\text{-}1} (\xi + N_t \equiv^0_{i\,t})$$

where $\equiv^0_{i\,t}$ is estimated from (12). In words, (13) says that investment is enough to cover depreciation, plus another term which is the net national rate of investment multiplied by a relative factor specific to state i. It is then possible to combine (13) with (10) to derive estimates of the capital stock for the years 1954 through 1976 in most states. (Lack of complete data for in earlier years pushes the first estimate for Alaska forward to 1962.)

In this way revised estimates k^1_{i76} are derived, and these can be used to restart the process by repeating steps (10) through (13). This results in successively revised estimates k^1_{it} and $\equiv^1_{it}$ for t=1977,…,1999; parameters $\forall^1_j$ and 2^1_q; v^1_{it} for t=54,…,76; and k^2_{i76}. This ends the first iteration.

This process can be repeated until either a convergence criterion is satisfied. The particular criterion used was an average absolute percentage change in the k_{i76} no greater than 10^{-10} between iterations.

The procedure was carried out for all 10 BEA categories of state and local government capital. Each of the ten equations converged in fewer than 10 iterations. The final estimates are shown in Table A4.

Table A4. Final Regression Results: Dependent Variable=Relative Investment Rate

	Equipment	Residential	Education	Hospital	Buildings nec
Iterations to Convergence	8	6	6	6	6
Final Regression Coefficients (p-values):					
Constant	-0.2590	0.5460	-0.0227	0.3663	0.5439
	(.0003)	(.0001)	(.8295)	(.0001)	(.0001)
Lagged relative population growth rates:					
Population lag 1	0.4337		0.3852		0.1336
	(.0001)		(.0001)		(.0001)
Population lag 2-5	0.1707	0.0662			
	0.0212	(.1225)			
Population lag 2-8			0.6865		0.0961
			(.0001)		(.0002)
Population lag 6-8		0.0805		0.1270	
		(.0532)		(.0009)	
State indicator variables:					
DVeryhi	5.6639	2.9842	7.2485	4.1282	1.7082
	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)
DHigh	1.2733	0.7862	1.6538	1.4240	1.3839
	(.0002)	(.0001)	(.0001)	(.0001)	(.0001)
DLow	-1.3392	-0.8119	-1.2254	-0.8407	-0.6383
	(.0001)	(.0001)	(.0003)	(.0001)	(.0001)
DVerylow				-1.7778	
				(.0001)	
Adjusted R^2	.432	.426	.311	.323	.402

Table A4. Continued

	Streets	C&D	Sewer	Water	Other
Iterations to Convergence	6	6	6	6	8
Final Regression Coefficients (p-values):					
Constant	0.8370	0.0938	0.4386	0.2036	0.2754
	(.0001)	(.0617)	(.0001)	(.0001)	(.0016)
Lagged relative population growth rates:					
Population lag 1				0.1967	0.2253
				(.0001)	(.0030)
Population lag 2		0.0950			
		(.0371)			
Population lag 2-5	0.2462				
	(.0001)				
Population lag 5			0.0516		
			(.1461)		
Population lag 2-8				0.4270	0.5368
				(.0001)	(.0001)
Population lag 3-8		0.2653			
		(.0001)			
Population lag 6-8	0.0770		0.0701		
	(.0318)		(.0594)		
State indicator variables:					
DVeryhi	4.955	2.387	1.348	2.270	13.405
	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)
DHigh	1.340	1.223	1.025	0.396	5.981
	(.0001)	(.0001)	(.0001)	(.0206)	(.0001)
DLow	-0.684	-0.785	-0.745	-0.126	-2.172
	(.0006)	(.0001)	(.0001)	(.0001)	(.0001)
Adjusted R^2	.502	.338	.268	.496	.528

The estimated pre-1977 investment series can be spliced onto the 1977-1999 data and the results used to estimate the average age of capital, by type, in each state. The procedure is as follows. First, set the average age of capital in state equal to the national average for 1953. Then, use perpetual accounting to recursively calculate the average age in subsequent years:

(14) $$a_{i\,t+1} = [(a_{i\,t}+1)\, k_{it}(1-\xi) + ½\, vn_{it+1}(1->)^6 + ap_t\, ve_{it+1}(1->)^6]/k^0_{i\,t+1}$$

where ap_t is the average age of the relevant type of private capital, in accord with the method used by BEA which assumes that existing assets purchased by governments are "typical".

The process of deriving estimating capital stock estimates for a particular local area begins by adapting the average age equation (14) to location m:

$$a_{mt} = [(a_{mt-1}+1)\, k_{mt-1}(1-\xi) + g_t\, v_{mt}(1->)^6]/[k_{mt-1}(1-\xi) + v_{mt}(1->)^6]$$

where $g_t = \dfrac{.5\sum_i vn_{it} + pa\sum_i ve_{it}}{\sum_i v_{it}}$, that is, the average end-of-the year age of total assets (including both new and used) purchased by all states in the country during the period.

Then (13) is substituted into the average age formula and the capital factor is eliminated in order to obtain

$$\text{(15)} \qquad a_{mt} = \frac{\left(a_{mt-1}+1\right)\left(1-\delta\right)+g_t\left(\delta+N_t\eta_{mt}\right)\left(1-\varepsilon\right)^6}{1-\delta+\left(\delta+N_t\eta_{mt}\right)\left(1-\varepsilon\right)^6}$$

Equation (13) can be used to estimate $\equiv_{mt}$ from local relative population growth factors Π_{mt}. Starting with the national average age for 1954 as initial estimate of the average age of the capital stock in m, (15) can be applied to calculate a_{mt} recursively for subsequent years.

The result is a recipe for estimating the age of the capital stock for a particular local area. To be implemented, the recipe requires only data on local population growth.

Given the age estimate—along with estimates of the parameters β_w, β_{wr}, and β_{wa} from the cost share equations, capital depreciation rates ξ_t from BEA, a current rate on tax-exempt bonds ϕ_{mt} , and values for w_{mt}, L_{mt}, and x_{mt} that can be obtained for any unit of government from data bases maintained by the U.S. Census Bureau—capital k_{mt} is the only unknown in the local cost share equation

$$\text{(16)} \quad [w_{mt}L_{mt} + x_{mt} + (\phi_{mt}+\xi_t)\,k_{mt}]\cdot[\beta_w + \beta_{wr}\ln((x_{mt}/k_{mt}+\phi_{mt}+\xi_t)/w_{mt}) + \beta_{wa}\,a_{mt} + \boldsymbol{\beta}_I'\boldsymbol{I}_{mt}] = w_{mt}L_{mt}$$

However, it's necessary to account for the fact that capital in (16) consists of both structures and equipment. Equations (7), (8), and (9) imply that

$$\text{(17)} \qquad k_{mt,s} = \gamma_{mt}k_{mt} \ \text{ and } \ k_{mt,e} = (1-\gamma_{mt})\,k_{mt} \ \text{ where}$$

$$\text{(18)} \qquad \gamma_{mt} = [1 + z(1-\xi_e)^{a_{mt,e}}(1-\xi_s)^{-a_{mt,s}}]^{-1}$$

By using the 1998 state average value (.11642) for z, it's possible to compute γ_{mt} from BEA's depreciation rates and the estimated ages of structures and equipment. In turn, γ_{mt} can be used to compute

$$\text{(19)} \qquad a_{mt} = a_{mt,s}\,k_{mt,s}/k_{mt} + a_{mt,e}\,k_{mt,e}/k_{mt} = \gamma_{mt}\,a_{mt,s} + (1-\gamma_{mt})\,a_{mt,e}$$

and

$$\text{(20)} \qquad \xi_{mt} = \gamma_{mt}\,\xi_{t,s} + (1-\gamma_{mt})\,\xi_{t,e}$$

for the blended age and depreciation rate of capital, respectively. Substitution into (16) yields a formula that can be applied in practice:

$$\text{(21)} \quad [w_{mt}L_{mt} + x_{mt} + (\phi_{mt} + \gamma_{mt}\xi_{t,s} + (1-\gamma_{mt})\,\xi_{t,e})\,k_{mt}]\cdot[\beta_w + \beta_{wr}\ln((x_{mt}/k_{mt} + \phi_{mt} + \gamma_{mt}\xi_{t,s} + (1-\gamma_{mt})\,\xi_{t,e})/w_{mt})] + \beta_{wa}(\gamma_{mt}\,a_{mt,s} + (1-\gamma_{mt})\,a_{mt,e}) + \boldsymbol{\beta}_I'\boldsymbol{I}_{mt}] = w_{mt}L_{mt}$$

This is the formula used to estimate k_{mt}, the dollar value of a particular type of government capital in a particular local area. Because capital appears twice in the nonlinear expression, a closed form solution for it does not exist. Finding the solution is a one-dimensional problem, however, so k_{mt} can be recovered through elementary numerical methods.